PSY 503: Foundations of Psychological Methods

Lecture 19: Heterogeneity and Experimental Regression Adjustments

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November 11, 2020

Heterogeneity

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- What happens if it's not?
- Let's open R Studio!

Why interaction terms?

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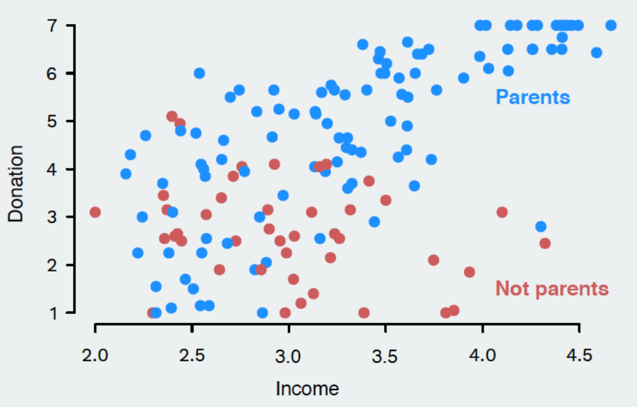
- Interaction terms will allow you to let the **slope** on one variable vary as a function of another variable
- Let's explore a different hypothetical dataset describing the relationship between income and donations conditional on having children

Let's see the data



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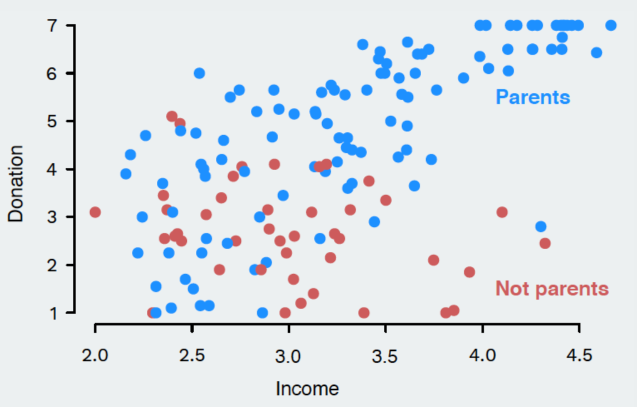
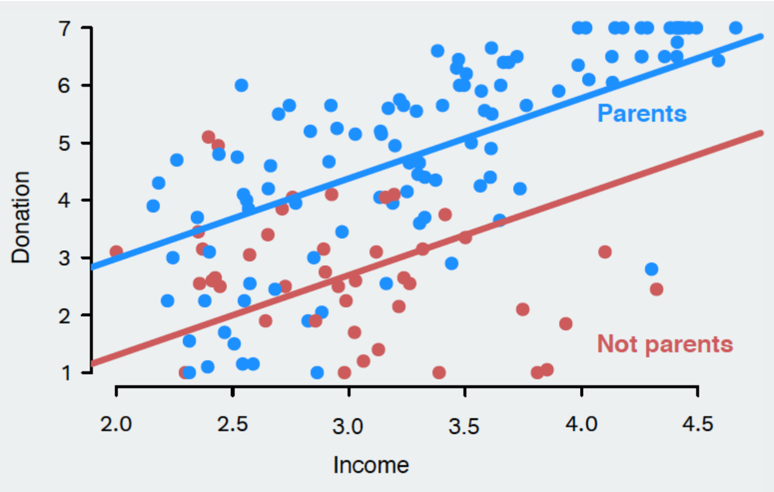


- Does it look like there's heterogeneity? What happens if we simply control for "parents" additively?

Controlling for "parent" additively



- The regression is a poor fit for non-parents. Can we allow for different slopes for each group?

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- We can add another covariate to the baseline model that allows the effect of income to vary by parental status
- This covariate is called an interaction term and it is the product of the two marginal variables of interest: $income \times parent$
- Here is the model with the interaction term:

$$\widehat{Y} = \widehat{\beta}_0 + \widehat{\beta}_1 X_1 + \widehat{\beta}_2 X_2 + \widehat{\beta}_3 X_1 X_2$$

Two lines in one regression

$$\widehat{Y}_i = \widehat{\beta}_0 + \widehat{\beta}_1 X_{1_i} + \widehat{\beta}_2 X_{2_i} + \widehat{\beta}_3 X_{1_i} X_{2_i}$$

- How can we interpret this model?

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- When $X_{2_i} = 0$:

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&= \widehat{\beta}_0 + \widehat{\beta}_1 X_{1_i} + \widehat{\beta}_2 \times 0 + \widehat{\beta}_3 X_{1_i} \times 0 \\
&= \widehat{\beta}_0 + \widehat{\beta}_1 X_{1_i}
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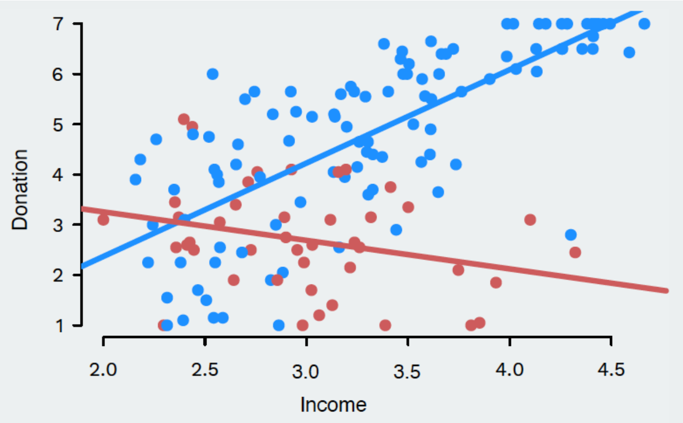
- When $X_{2_i} = 1$:

$$\begin{aligned}\widehat{Y}_i &= \widehat{\beta}_0 + \widehat{\beta}_1 X_{1_i} + \widehat{\beta}_2 X_{2_i} + \widehat{\beta}_3 X_{1_i} X_{2_i} \\ &= \widehat{\beta}_0 + \widehat{\beta}_1 X_{1_i} + \widehat{\beta}_2 \times 1 + \widehat{\beta}_3 X_{1_i} \times 1 \\ &= (\widehat{\beta}_0 + \widehat{\beta}_2) + (\widehat{\beta}_1 + \widehat{\beta}_3) X_{1_i}\end{aligned}$$

Example interpretation of the coefficients

	Intercept for X_{2i}	Slope for X_{2i}
$X_{2i} = 0$	$\widehat{\beta}_0$	$\widehat{\beta}_1$
$X_{2i} = 1$	$\widehat{\beta}_0 + \widehat{\beta}_2$	$\widehat{\beta}_1 + \widehat{\beta}_3$



General interpretation of the coefficients

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- $\widehat{\beta}_2$: average difference in Y_i between $X_{2_i} = 1$ group and $X_{2_i} = 0$ group when $X_{1_i} = 0$
- $\widehat{\beta}_3$: change in the effect of X_{1_i} on Y_i between $X_{2_i} = 1$ group and $X_{2_i} = 0$ group

Lower order terms

- **Principle of Marginality**: Always include the lower order terms!
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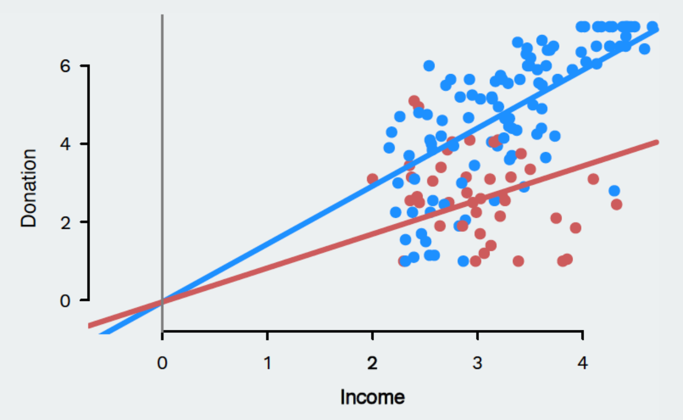

Omitting lower order terms

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- This model does not allow for a difference between parents and non-parents when income is 0
  - This distorts slope estimates
  - Very rarely justified
  - Yet, for some reasons, people do it...

A note on interactions with non-binary discrete and continuous variables

- Same principle!
- Plug in values in the equation to get the marginal effect of X_{1_i} conditional on specific values of X_{i_2}

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- Do not interpret the coefficients on the lower order terms as marginal effects
  - They give the marginal effect only for the case where the other variable is equal to zero
- The p-value on the interaction term can be used as a test against the null of no interaction, but significant tests for the lower order term rarely make sense

Polynomials

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- A third order polynomial is given by:
  $Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 X_i^3 + u_i$

Polynomial example: Income and Age

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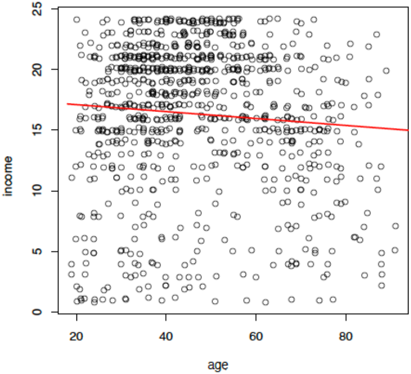
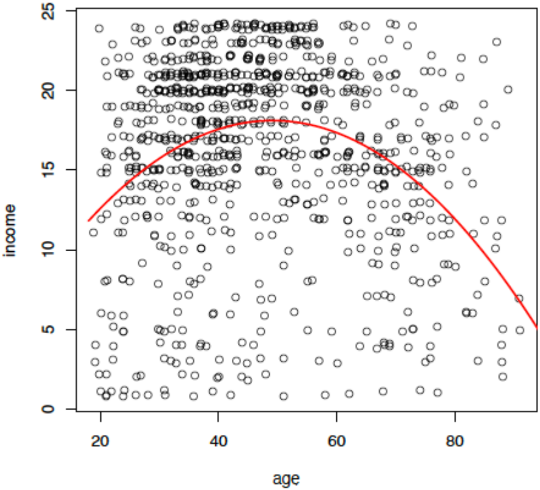
- Here we can see that simple linear regression does not seem to fit the data

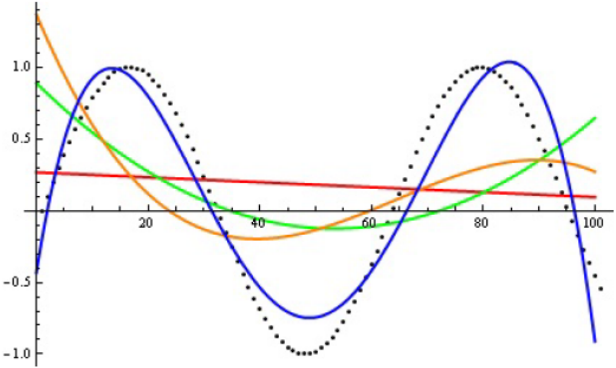


Polynomial example: Income and Age

- A second order polynomial in age fits the data a lot better:



Higher order polynomials



Covariate Adjustments in Experimental Designs

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- Make sure to pre-register your model!

Including covariates in regression analysis

Let's open R studio!

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  - How? Use Monte-Carlo simulation...

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  - i.e., the schedule of covariates is FIXED regardless of whether an individual was assigned to treatment vs. control condition
- **IMPLICATION: DO NOT adjust for posttreatment covariates!**
  - It will bias ATE
  - It will turn your experimental study into a correlational study
  - It will not allow you to make **any** causal claims

What is a posttreatment covariate?

- Any variables that: were measured after the treatment / may be affected by the treatment

Why you SHOULD NOT adjust for posttreatment covariate?

- Adjusting for posttreatment covariates **introduces bias** in your estimate of the average treatment effect, *even if treatment was randomly assigned to participants*
  - That is because variables that are affected by treatment cannot possibly be independent of treatment assignment
- Pervasive issue in social psychology —so called mediation analysis!

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Back to R Studio!