PSY 503: Foundations of Psychological Methods

Lecture 19: Heterogeneity and Experimental Regression Adjustments

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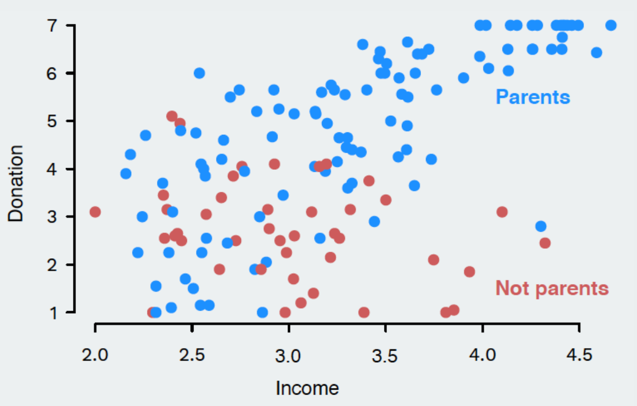
Heterogeneity

- So far, we have considered cases in which β_1 was constant across subgroups
- What happens if it's not?
- Let's open R Studio!

Why interaction terms?

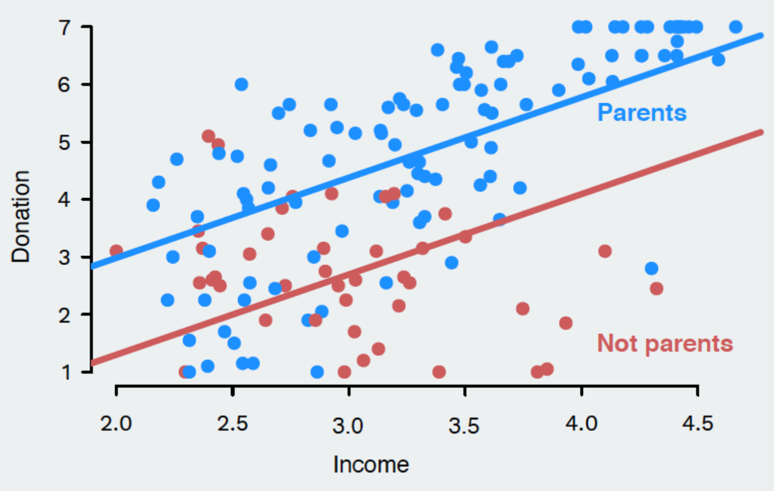
- Interaction terms will allow you to let the **slope** on one variable vary as a function of another variable
- Let's explore a different hypothetical dataset describing the relationship between income and donations conditional on having children

Let's see the data



- Does it look like there's heterogeneity? What happens if we simply control for "parents" additively?

Controlling for "parent" additively



- The regression is a poor fit for non-parents. Can we allow for different slopes for each group?

Interactions with a binary variable

- Let X_2 be binary
- In this case, $X_2 = 1$ for parents
- We can add another covariate to the baseline model that allows the effect of income to vary by parental status
- This covariate is called an interaction term and it is the product of the two marginal variables of interest: $income \times parent$
- Here is the model with the interaction term:

$$\widehat{Y} = \widehat{\beta}_0 + \widehat{\beta}_1 X_1 + \widehat{\beta}_2 X_2 + \widehat{\beta}_3 X_1 X_2$$

Two lines in one regression

$$\widehat{Y_i} = \widehat{\beta}_0 + \widehat{\beta}_1 X_{1_i} + \widehat{\beta}_2 X_{2_i} + \widehat{\beta}_3 X_{1_i} X_{2_i}$$

- How can we interpret this model? We can plug in the two possible values of X_{2i}
- When $X_{2_i} = 0$:

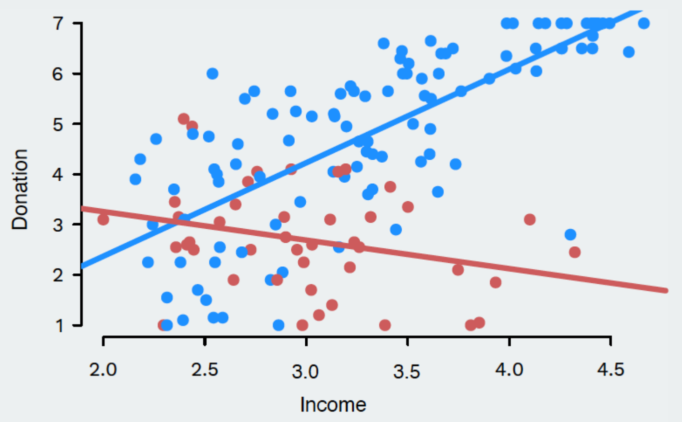
$$\begin{aligned} \widehat{Y_i} &= \widehat{\beta}_0 + \widehat{\beta}_1 X_{1_i} + \widehat{\beta}_2 X_{2_i} + \widehat{\beta}_3 X_{1_i} X_{2_i} \\ &= \widehat{\beta}_0 + \widehat{\beta}_1 X_{1_i} + \widehat{\beta}_2 \times 0 + \widehat{\beta}_3 X_{1_i} \times 0 \\ &= \widehat{\beta}_0 + \widehat{\beta}_1 X_{1_i} \end{aligned}$$

- When $X_{2_i} = 1$:

$$\begin{aligned} \widehat{Y_i} &= \widehat{\beta}_0 + \widehat{\beta}_1 X_{1_i} + \widehat{\beta}_2 X_{2_i} + \widehat{\beta}_3 X_{1_i} X_{2_i} \\ &= \widehat{\beta}_0 + \widehat{\beta}_1 X_{1_i} + \widehat{\beta}_2 \times 1 + \widehat{\beta}_3 X_{1_i} \times 1 \\ &= (\widehat{\beta}_0 + \widehat{\beta}_2) + (\widehat{\beta}_1 + \widehat{\beta}_3) X_{1_i} \end{aligned}$$

Example interpretation of the coefficients

	Intercept for X_{2i}	Slope for X_{2i}
$X_{2i} = 0$	$\widehat{\beta}_0$	$\widehat{\beta}_1$
$X_{2i} = 1$	$\widehat{\beta}_0 + \widehat{\beta}_2$	$\widehat{\beta}_1 + \widehat{\beta}_3$

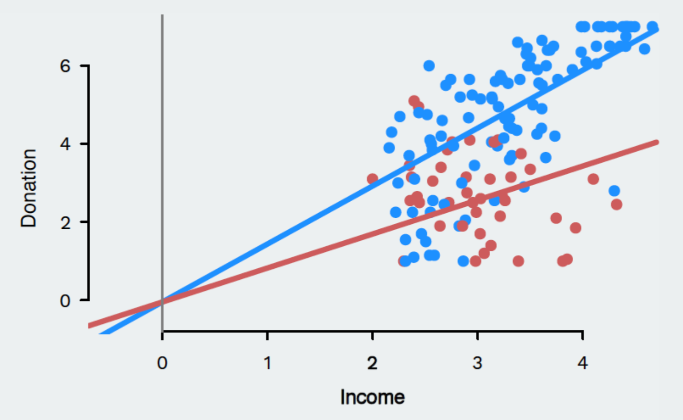


General interpretation of the coefficients

- $\widehat{\beta}_0$: average value of Y_i when both X_{1_i} and X_{2_i} are equal to 0
- $\widehat{\beta}_1$: a one-unit change in X_{1_i} is associated with a $\widehat{\beta}_1$-unit change in Y_i when $X_{2_i} = 0$
- $\widehat{\beta}_2$: average difference in Y_i between $X_{2_i} = 1$ group and $X_{2_i} = 0$ group when $X_{1_i} = 0$
- $\widehat{\beta}_3$: change in the effect of X_{1_i} on Y_i between $X_{2_i} = 1$ group and $X_{2_i} = 0$ group

Lower order terms

- **Principle of Marginality**: Always include the lower order terms!
- Imagine that we ommitted the lower order term:



Omitting lower order terms

$$\widehat{Y}_i = \widehat{\beta}_0 + \widehat{\beta}_1 X_{1_i} + 0 \times X_{2_i} + \widehat{\beta}_3 X_{1_i} X_{2_i}$$

- This model does not allow for a difference between parents and non-parents when income is 0
  - This distorts slope estimates
  - Very rarely justified
  - Yet, for some reasons, people do it...

A note on interactions with non-binary discrete and continuous variables

- Same principle!
- Plug in values in the equation to get the marginal effect of X_{1_i} conditional on specific values of X_{i_2}

Summary for interactions

- Do not omit lower order terms because it usually imposes unrealistic restrictions
- Do not interpret the coefficients on the lower order terms as marginal effects
  - They give the marginal effect only for the case where the other variable is equal to zero
- The p-value on the interaction term can be used as a test against the null of no interaction, but significant tests for the lower order term rarely make sense

Polynomials

Polynomial terms

- Polynomial terms are a special case of the continuous variable interactions
  - where $X_1 = X_2$

$$\begin{aligned}\widehat{Y}_i &= \widehat{\beta}_0 + \widehat{\beta}_1 X_{1_i} + \widehat{\beta}_2 X_{1_i} + \widehat{\beta}_3 X_{1_i} X_{1_i} \\ &= \widehat{\beta}_0 + (\widehat{\beta}_1 + \widehat{\beta}_2) X_{1_i} + \widehat{\beta}_3 X_{1_i} X_{1_i} \\ &= \widehat{\beta}_0 + (\widehat{\beta}_1 + \widehat{\beta}_2) X_{1_i} + \widehat{\beta}_3 X_{1_i}^2\end{aligned}$$

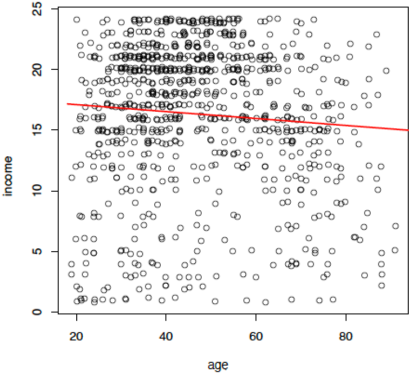
- This is called a **second order polynomial** in X_1, which we generally write:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + u_i$$

- A third order polynomial is given by: $Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 X_i^3 + u_i$

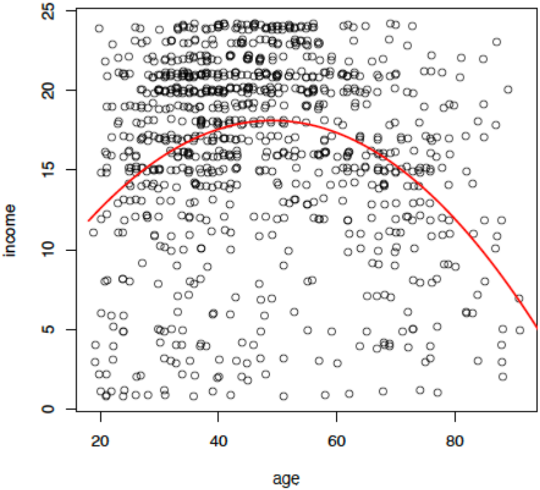
Polynomial example: Income and Age

- Classic example of a non-linear relationship between two variables: income and age
- Here we can see that simple linear regression does not seem to fit the data

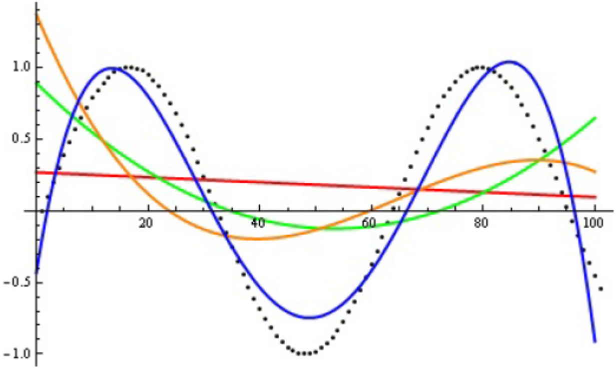


Polynomial example: Income and Age

- A second order polynomial in age fits the data a lot better:



Higher order polynomials



Covariate Adjustments in Experimental Designs

Why include PRETREATMENT covariates in experimental data analysis?

- In experiments: Regression analysis with and without PRETREATMENT covariate adjustments is unbiased!
  - Random assignment to conditions creates two groups that are, in expectation, identical prior to treatment. This implies pretreatment covariates are not correlated with the treatment
- So why adjust for covariates? Controlling for pretreatment covariates:
  - Increases precision
  - Increases statistical power
- Make sure to pre-register your model!

Including covariates in regression analysis

Let's open R studio!

Any drawbacks to adjusting for pretreatment covariates?

- In small sample ($n < 20$): Including pretreatment covariates introduces bias in the estimate of the ATE
- In larger samples ($n > 20$): Bias becomes negligible
- If you are going to include more than one pretreatment covariates, make sure sample is large enough!
  - How? Use Monte-Carlo simulation...

Including covariates in regression analysis

- **Key assumption:** Covariates are unaffected by treatment assignment
  - i.e., the schedule of covariates is FIXED regardless of whether an individual was assigned to treatment vs. control condition
- **IMPLICATION: DO NOT adjust for posttreatment covariates!**
  - It will bias ATE
  - It will turn your experimental study into a correlational study
  - It will not allow you to make **any** causal claims

What is a posttreatment covariate?

- Any variables that: were measured after the treatment / may be affected by the treatment

Why you SHOULD NOT adjust for posttreatment covariate?

- Adjusting for posttreatment covariates **introduces bias** in your estimate of the average treatment effect, *even if treatment was randomly assigned to participants*
  - That is because variables that are affected by treatment cannot possibly be independent of treatment assignment
- Pervasive issue in social psychology —so called mediation analysis!

Why you SHOULD NOT adjust for posttreatment covariate?

Back to R Studio!